CE201 MATHEMATICS

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Lecturers:	Dr. N. Adams (Autumn term) Dr. A.G. Walton, (Autumn and Spring terms) Dr. A.O. Gogolin, (Spring and Summer terms)
Structure:	56 lecture hours
Links:	Techniques useful various modules involving Hydrodynamics & Structures

Introduction

All engineers need to understand and use a variety of mathematical, computational and statistical methods in order to deal with quantitative problems. Civil engineers, in particular, require a knowledge of linear algebra in order to understand the statics and dynamics of structures. Complex variable theory is a powerful tool for gaining insight into solutions of Laplace's equation which governs many fluid flows. Fourier series is used for representing various signals and for solving partial differential equations such as the diffusion equation. Statistics and probability are necessary in order to understand random variations in physical phenomena and to make practical decisions in the light of collected data. These and other mathematical methods are introduced and their practical importance is discussed and illustrated by example.

Aims

The aim of the mathematical methods part of the module is to provide the student with an insight into various techniques that will prove useful in modules in the second and subsequent years of the course. These techniques include the understanding and use of Fourier series and its connection with waves and vibrations and the use of complex variable theory with particular application to Laplace's equation and various fluid flows. The module provides an insight into the properties of functions of position in three-dimensional space and their application to mechanical problems. There is extensive study of linear algebra with application to static and dynamical properties of structures. In addition various methods for solving important ordinary and partial differential equations are discussed.

Module Structure

There are two lectures per week in the first term, three per week in the second term and one per week in the third term. The main methods of teaching are by blackboard and overhead projector. Some printed material is issued but students are also expected to make their own notes. In addition to the lectures there is one tutorial per week. This is an opportunity for the students to ask the lecturer or his assistants about the module and to work through problem sheets that have been issued.

Syllabus

O D E 's: Picard's method. Variation of one parameter. Systems of ODE's with constant coefficients; matrix form leading to eigenvalue problem, coupled oscillations.

Numerical Methods: First and second order ODE's: Euler's method and simple Runge-Kutta method for initial value problems. Error analysis.

Fourier Series: Standard formulae; even and odd functions; sine and cosine series; half-range series. Parseval's theorem. The Gibbs phenomenon.

P D E 's: Second order PDE's: classification; Laplace, wave and diffusion equations; d'Alembert's solution of the one-dimensional wave equation; separation of variables including the use of Fourier series.

Vector Calculus: Vectorial differentiation; grad, div and curl operators.

Complex Variables: Complex variable theory; analyticity; Cauchy-Riemann equations; conjugate functions; orthogonal functions.

Integration: Line integrals. Double integrals; inversion of order of integration; mappings; Jacobian; change of variable.

Linear Algebra: Linear algebraic equations: Cramer's rule; Gaussian elimination; LU factorisation. Iterative method for solving $A\mathbf{x} = \mathbf{b}$; Gauss-Seidel method. Consistency conditions; rank. Eigenvalues and eigenvectors; linear independence of eigenvectors; diagonalisation; powers of a matrix; Cayley-Hamilton theorem for distinct eigenvalues case.

Probability And Statistics: Random variation and frequency distributions; population and sample; probability and probability distributions. Sample spaces and events; addition law of probability; conditional probability and independence; law of total probability and Bayes's theorem; examples. Expected value; variance; covariance. Uniform, geometric, binomial, Poisson, exponential and normal distributions. Sampling distribution; confidence interval for a mean and for the difference of two means; test of significance for a single mean and for the difference between two means; relation between confidence interval and significance test; paired observations. Linear statistical models; method of least squares; linear regression; multiple regression. Reliability: failure rate; hazard function; estimating the expected time to failure and the hazard function; the reliability of devices in series and parallel. Parameter estimation by method of moments and maximum likelihood.

Course Topics (Chronological order)

	TOPIC	No. Of Lectures	Term
1	Ordinary differential equations	5	Aut
2	Fourier series	7	Aut
3	Partial differential equations	6	Aut
4	vector calculus	2	Aut
5	Probability and statistics	18	Spr
6	Complex variables	3	Spr
7	Linear algebra	7	Spr
8	Integration	6	Spr/Sum

Notes:
(i) Matrix form of ODE's, coupled oscillations, to follow Linear Algebra;
(ii) Double integrals covered in 4 lectures in Summer term.

Tutorial arrangements
One hour, once per week for the entire year.
Total of 15 problem sheets, issued at regular intervals

Assessment
One 3-hour and one 2-hour written examination at the end of session (Part II, Papers 3 and 4).

Paper 3 consists of 12 questions with rubric " Answer eight questions ".

Paper 4 to consist of 6 questions, 1 question on linear algebra and 5 on statistics. The question on linear algebra to be compulsory. Rubric "Answer Question 1 and any three other questions ".

Department of Mathematics Formulae Sheet is distributed with both papers and appropriate statistical tables with Paper 4.

Progress Tests
One hour progress tests in December and in March.

Recommended Textbooks/Reading:
JEFFREY, A., Mathematics for Engineers and Scientists.
STEPHENSON, G., Mathematical Methods for Science Students, Longmans, 1973.
KREYSZIG, E., Advanced Engineering Mathematics (8th Edition), John Wiley & Sons, 1999.
DEVORE, Probability and Statistics for Engineering and the Sciences.

ANG and TANG, Probability Concepts in Engineering, Planning and Design, Vol. 1.
BENJAMIN and CORNELL, Probability, Statistics and Decisions for Civil Engineers.
MENDENHALL and SINCICH, Statistics for Engineering and the Sciences.

Learning Outcomes

At the end of the module a students are expected to be able to:

1. determine the Fourier series expansions of the simple functions;
2. use the Cauchy-Riemann equations; determine orthogonal trajectories;
3. understand simple graphical and numerical representations of data;
4. determine probabilities for events using basic laws of probability;
5. calculate the expectation and variance of random variables;
6. perform hypothesis tests and construct confidence intervals for the man of a random sample;
7. carry out calculations involving the reliability of devices;
8. evaluate double integrals by change of variables and by inverting the order of integration;
9. carry out calculations involving the grad, div and curl vector operators;
10. solve systems of linear algebraic equations by Gaussian elimination, by LU factorization and by the Gauss-Seidel iterative method;
11. find the eigenvalues and eigenvectors of a square matrix; apply the Cayley-Hamilton theorem;
12. find the general solution of linear second order ordinary differential equations by the method of variation of one parameter; solve ODE's in matrix form;
13. use Euler's and simple Runge-Kutta methods to find numerical solutions to initial value problems for first and second order ODE's; carry out an error analysis of these methods;
14. classify second order linear partial differential equations; derive d'Alembert's solution on the one-dimensional wave equation; derive solutions of certain partial differential equations by separation of variables using Fourier series.