## CE102 MATHEMATICS

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Lecturers: Autumn/Spring terms: Dr C.J. Ridler-Rowe (Rm 647 Huxley Bldg) Spring/Summer terms: Professor J.R. Cash (Rm 6M40 Huxley Bldg)

Structure: 50 lectures and 50 tutorials over 3 terms
Links: CE201, and various modules involving hydrodynamics \& structures.

## Introduction

A solid grounding in Mathematics is essential to the training of an engineer. Mathematics provides a model for describing the physical world around us and, with its aid, engineers are able to gain insights into the behaviour of materials and processes which are necessary for design and analysis. The computer - as a tool both for gathering and processing data and for predicting behaviour - increasingly demands a deeper understanding of mathematical techniques. You will study Mathematics in the first and second years of your course. The aim of the first year module is to draw you all to a common level of understanding of the mathematical methods needed in other subjects and to provide a basis for more advanced studies in the second year.

## Aims

To introduce mathematics as a logical and structured discipline; to ensure that all students acquire the mathematical knowledge and skills required for their first year civil engineering courses; to provide a basis for the more advanced mathematical techniques which are required in later years of the course.

## Syllabus

Analysis: Functions of one variable : odd, even, inverse functions. Limits : continuous and discontinuous functions. Differentiation : continuity and differentiability ; implicit and logarithmic differentiation ; Leibniz's formula ; stationary points and points of inflection ; curve sketching ; polar coordinates. Mean value theorem ; Taylor's and Maclaurin's series ; l'Hopital's rule. Convergence of power series ; ratio test ; radius of convergence. Complex numbers : the complex plane ; polar representation ; de Moivre's theorem ; In z and $\exp (z)$. Hyperbolic functions: inverse functions ; series expansions ; relations between hyperbolic and trigonometric functions. Integration : definite and indefinite integrals ; the fundamental theorem ; improper integrals ; integration by substitution and by parts ; partial fractions ; applications. Functions of more than one variable : partial differentiation ; total differentials ; change of variable ; Taylor's theorem for a function of two variables ; stationary points ; contours.

Linear Algebra : Vector algebra : basic rules ; cartesian coordinates ; scalar and vector products ; applications to geometry ; equations of lines and planes ; triple products ; linear dependence. Matrix algebra : double suffix notation ; basic rules ; transpose, symmetric, diagonal, unit, triangular, inverse and orthogonal matrices. Determinants : basic properties ; Cramer's rule. Linear algebraic equations :
consistency ; linear dependence ; Gauss-Jordan method ; Gaussian elimination ; LU factorisation.

Ordinary Differential Equations : First order equations : separable, homogeneous, exact, linear. Second order linear equations with constant coefficients.

Numerical Methods : Fixed Point and Newton-Raphson iterative methods for the solution of non-linear algebraic equations. Numerical integration : trapezium and Simpson's rules ; Richardson extrapolation.

Course Analysis ( Chronological order )

| TOPIC |  | NUMBER OF LECTURES |
| :--- | :--- | :--- |
| 1 | Functions of one variable | 3 |
| 2 | Limits of functions | 3 |
| 3 | Differentiation | 3 |
| 4 | Taylor and Maclaurin series | 3 |
| 5 | Linear algebra | 10 |
| 6 | Partial differentiation | 4 |
| 7 | Complex numbers | 6 |
| 8 | Integration | 6 |
| 9 | Ordinary differential equations | 7 |
| 10 | Numerical Methods | 5 |

## Tutorial Arrangements

(i) 3 classes, each meeting twice a week
(ii) A total of 16 problem sheets, issued at regular intervals.

## Assessment

Two 3-hour written examinations at the end of session (Part I, Papers 1 and 2 ) each containing 10 questions. Rubric "Answer eight questions". Department of Mathematics Formulae Sheet is distributed with the examination papers.

## Non-Assessed Coursework

Diagnostic test 1 in October selects students to attend PMC; diagnostic test 2 taken at the end of PMC. Test in December, and then in March for weak students. Coursework marked weekly.

## Recommended Texts

FAIRES J D and BURDEN J D: Numerical Methods, ITP, 1993
STEPHENSON G: Mathematical Methods for Science Students, Longmans, 1973
STROUD K A: Engineering Mathematics, MacMillan, 1995

## Learning Outcomes

By the end of the course, students will be able to :
(i) apply the techniques of the differential calculus to determine the maxima and minima of functions and to curve sketching ;
(ii) determine the Maclaurin series expansions of the standard functions and to apply the ratio test to check for convergence;
(iii) carry out simple calculations in vector algebra, vector geometry and matrix algebra;
(iv) find the inverse of matrices by the Gauss-Jordan method and the solution of systems of linear algebraic equations by Gaussian elimination and by LU factorisation;
(v) carry out calculations involving the differentiation of functions of two variables; apply Taylor's theorem for a function of two variables in the derivation of contour maps;
(vi) manipulate complex numbers in both standard and polar forms; apply de Moivre's theorem;
(vii) carry out calculations involving the hyperbolic functions and relate these functions to the trigonometric functions;
(viii) integrate simple functions using the methods of substitution, integration by parts and reduction formulae;
(ix) find analytic solutions of certain first order ordinary differential equations ; find the general solution of certain linear constant coefficient second order ODE's;
(x) apply fixed-point and the Newton-Raphson iterative methods to determine solutions of non-linear algebraic equations ; apply the trapezium and Simpson's rules to evaluate numerically integrals of simple functions ; apply Richardson extrapolation.

